

On the Mathematical Foundations of Crash-Proof Syntax

Crash-proof Syntax (Frampton and Gutmann 1999, 2002, henceforth CPS) is a theory of grammar in which the computational system of syntax is optimally designed, for instance by means of self-organization of internal systems throughout adaptation, so that only grammatical outputs can be derived and be interpreted at the interface levels, without recourse to output filters or evaluation metrics that compare between derivations. We discuss two related syntactic relations that are central to the components of CPS, i.e. (i) the *local dependency* relation between functional and lexical categories that leads to respective *Probe-Goal* valuation (Chomsky 2001), and (ii) the projection of heads as the *label* of constituent structures following the definition of Merge [1] (Chomsky 1995). The first relation, coupled by cyclic computations in CPS, is rephrased as the *Locus Principle* (also in Collins 2002) which states that each cycle is triggered by an introduction of a new head, and the selector feature (e.g. EPP-feature, phi-features) of that head has to be satisfied before a new cycle (i.e. another lexical item) is introduced. For the second relation, the label of a constituent structure predetermines its category and the particular dependency relation with an incoming head, e.g. a DP merges with a V. We argue that the two relations can find full parallels in the *basic number theory* (cf. Hauser et al. 2002), which provides some mathematical foundations for the thesis of CPS.

We focus on *Peano Arithmetic* (PA) as the foundations of mathematical axioms (Rosen et al. 1999). PA consists of a list of statements, the most important of which concerns the *successor function* S that applies to natural numbers [2]. S can also be set-theoretically defined, e.g. by *Zermelo–Fraenkel* (Z-F) *Axioms* [3][4]. Central to PA is a finite set of natural numbers as *discrete objects* whose mutual relations can only be defined *locally*. Natural number 3 is defined by $S(2)$, or a Z-F set $\{2, \{2\}\}$, which is subsequently defined by natural number 1, and so on (i.e. $\{0, 1, 2\}$). Along this idea, for instance, there is no way that 7 can be defined ‘long-distance’ by 3 and skips the intervening 4, 5 and 6, i.e. natural numbers are only contextually and moreover locally defined by their adjacent members. A number n is defined by its adjacent $n-1$, whereas at the same time it defines the adjacent $n+1$. From this we understand that natural number has *two* roles, one providing a complex expression defined by simpler objects, whereas another helping define a more complex expression. These relations are analogous to the definition of cycles in CPS in which a new cycle (cf. a larger number) cannot be introduced if the current cycle (cf. a smaller number) is not ‘saturated’ (cf. defined), in the sense that the selector feature of the current head is not valued or checked off. While in PA the contextual relation is merely defined by the successor function S , in syntax it ranges over *theta-role assignment*, *subcategorization*, and *EPP*, all of which are *local relations*. Such mathematical foundation also hinges on labels. CPS assumes that the category that bears more selector features always projects its label, e.g. the EPP-feature of T leads to the projection of TP. An analogy is found in PA that a complex structure is ‘labeled’ by a simpler structure that it is minimally local with. Thus 6 is the ‘label’ of 7 since the latter is set-theoretically defined as $6 \cup \{6\}$, and so on. While we do not claim that all algorithms worked out in the Minimalist Program and CPS find their roots in PA (e.g. agreement is largely domain-specific), we should at least understand the way in which syntax as a formal system can receive an alternative if not better understanding based on solid mathematical foundations.

Definitions:

[1] Merge $\alpha, \beta \rightarrow \{\gamma, \{\alpha, \beta\}\}$, where γ is the label of constituent projected by either α or β .

[2] Peano Arithmetic is a structure $(N, 0, S)$ with the following axioms:

(i) 0 is a natural number.

(ii) For every natural number $n \in N$, $S(n)$ is a natural number.

(iii) 0 is not the successor of any natural number;

(iv) two different natural numbers cannot have the same successor.

[3] $S(a) = a \cup \{a\}$

Define $0 = \{\emptyset\}$, $1 = S(0) = S\{\emptyset\}$, etc.

[4] Addition is the function $+: N \times N \rightarrow N$, defined recursively as:

$a + 0 = a$,

$a + (S(b)) = S(a + b)$

For example,

$a + 1 = a + S(0) = S(a + 0) = S(a)$

References:

Chomsky, Noam. 1995. *The Minimalist Program*. Cambridge, Mass: MIT Press.

Chomsky, Noam. 2001. "Derivation by phase.", in M. Kenstowicz (ed.), *Ken Hale: a Life in Language*. Cambridge, Mass: MIT Press. 1-52.

Collins, Chris. 2002. "Eliminating labels.", in Sam D. Epstein and T. Daniel Seely (eds.), *Derivation and Explanation in the Minimalist Program*. Oxford: Blackwell. 42-61.

Frampton, John and Sam Gutmann. 1999. "Cyclic computation, a computationally efficient minimalist syntax.", *Syntax* 2: 1-27.

Frampton, John and Sam Gutmann. 2002. "Crash-proof syntax.", in Sam D. Epstein and T. Daniel Seely (eds.), *Derivation and Explanation in the Minimalist Program*. Oxford: Blackwell. 90-105.

Hauser, Marc D, Noam Chomsky and W. Tecumseh Fitch. 2002. "The faculty of language: what is it, who has it and how did it evolve?", *Science* 298: 1569-1579.

Rosen, Kenneth H. ed. 1999. *Handbook of Discrete and Combinatorial Mathematics*. Boca Raton, CRC Press.